Fermi National Accelerator Laboratory

FERMILAB-PUB-86/169-T
December, 1986

Dynamical Gauge Hierarchies

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Abstract

A program is proposed to construct dynamical models which produce gauge boson hierarchies without fine tuning. The low energy physics is described by the global symmetry group G, a coset manifold G/H representing Goldstone modes and a "superweak" gauge group $H_w \subset G$. The essential feature of these models is that H_w can not be embedded into the unbroken subgroup H ,*i.e.*$H_w \not\subset H$. It is argued that the effective potential induced by the "superweak" interactions can reproduce a Higgs potential with an intrinsically small parameter "Higgs mass"/"Higgs self coupling" $\sim O(\alpha_w)$. The salient features of the program are illustrated in a model with $G/H = SU(6)/Sp(6)$ and $H_w = SU(2) \not\subset Sp(6)$. In particular it is shown that a nontrivial Higgs self interaction (without mass term) can be generated in the leading order of "superweak" interactions.

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Operated by Universities Research Association Inc. under contract with the United States Department of Energy

The origin of the Fermi scale seems to be the key to understanding the physics underlying the standard model. The attempts to unravel its nature is compounded by the fact that not only are the masses of weak vector bosons generated at that scale but also those of fermions. The initial enthusiasm for the technicolor origin of the Fermi scale [1,2] has mainly subsided because it does not address this second issue.The extended technicolor models [3,4] have not met the challenge of explaining the fermion mass spectrum successfully. The situation calls for an alternative dynamical framework for generating the Fermi scale.

There seems to be ample evidence to believe that the fermion masses are generated sequentially *i.e.* "fed down" from higher to lower generations via superheavy gauge boson exchanges (see *e.g.* Ref. [5]).The "feed down" mechanism may be viable in the dynamical framework provided the composite Higgs behaves as an elementary object throughout the energy range extending to the mass scale (Λ) of superheavy gauge bosons. This suggests that the composite Higgs is a pseudo-Goldstone boson of a fundamental dynamics which is the source of the superheavy gauge bosons.The approach pursued in this note incorporates this essential feature naturally.

The idea of the composite Higgs as a pseudo-Goldstone boson was originally advanced by Georgi et al.[6].These authors have argued that to the lowest order in gauge couplings the effective potential induced by an appropriately chosen superweak interactions can produce a Higgs condensate. Unfortunately,in such approach an adequate Fermi scale Λ_F and a dynamical hierarchy $\Lambda_F \ll \Lambda$ results only if the gauge couplings are fine tuned. In this note a general program will be proposed to identify models which can produce dynamical hierarchies without this fine tuning. Furthermore, a specific model will be constructed to exhibit some of the important features of the general program.

It will be assumed that the low energy models considered below arise from an as yet unknown strong dynamics of fermions and gauge bosons. The models in question are described by a global symmetry group G which is broken down dynamically to $H \subset G$ and also explicitly broken down to a subgroup of G containing the gauge group $H_w \subset G$ of superweak interactions [7]. The embedding $H_w \subset G$ defined up to conjugacy $\ell H_w \ell^{-1}, \ell \in G/H$ should be fixed dynamically, i.e., by minimizing the effective potential $V(\ell)$ generated by H_w-interactions in various orders of its fine structure constant $\alpha_w = g_w^2/4\pi$ [8-11].

The effective potential $V(\ell(x)) = V(x)$ can be viewed as a function defined on the coset manifold G/H with coordinates $\{x^\mu\}$ representing would-be Goldstone modes of the strong dynamics. In general, the minimum $x^\mu = \xi^\mu$ of $V(x)$ is determined in two stages. First, one identifies the submanifold $M \subset G/H$ on which the leading component $V^{(1)}(x) \sim \alpha_w$ of $V(x)$ attains its minimum. Next, one identifies $\xi \in M$ by minimizing on M the higher order component $V^{(2)}(x) \sim O(\alpha_w)$ of $V(x)$. The broken generators of the "aligned" gauge group $H_w^\xi = \ell(\xi)\, H_w \ell^{-1}(\xi)$ are sources of superweak gauge bosons with masses $M_w \sim g_w \Lambda$ whereas unbroken generators form the surviving gauge subgroup $H_w' \subset H_w^\xi$.

We are now in a position to state the necessary conditions for the breakdown of the residual symmetry H_w' and generation of a dynamical hierarchy with a new mass scale $M_w' \ll M_w$. Consider fluctuations $\phi = \left\{x^{\hat\mu} - \xi^\mu\right\}$ about the minimum $\{\xi^\mu\}$ in a direction $\hat\mu$ which is not neutral under H_w'. The associated would be GB mode will break the gauge symmetry H_w' provided it condenses,*i.e.*

$$V^{(1)}(\phi) = a\phi^{2n} + O\left(\phi^{2n+2}\right), \tag{1}$$

$$V^{(2)}(\phi) = -b\phi^{2m} + O\left(\phi^{2m+2}\right), \tag{2}$$

where

$$a \gg b > O\ , \text{ and } 1 \leq \mathrm{m} < \mathrm{n}. \tag{3}$$

Here $a \sim \alpha_w$ and normally $b \sim \alpha_w^2 ln\alpha_w$ (See e.g. Ref [11]). Under the above conditions, some H_w' gauge boson is rendered massive and a dynamical hierarchy results:

$$M_w'^2 \sim g_w^2 \Lambda^2 (b/a)^{\frac{1}{n-m}} \ll g_w^2 \Lambda^2 \sim M_w^2 \tag{4}$$

Two important remarks are in order. First, the absence of quadratic fluctuations in Eq.(1) imposes a stringent constraint on the models in question. Indeed it demands that the gauge group H_w can not be embedded into H, *i.e.* $gH_w g^{-1} \not\subset H, g \in G$.Otherwise the generators which are the sources of such fluctuations would commute with H_w^ξ and fluctuations of all orders would be absent. Secondly, the extremum $x^\mu = \xi^\mu \in M$ is required to be a saddle point of $V^{(2)}(x)$ since it is by definition a minimum for deviations tangential to M and according to Eqs.(2,3) a maximum for normal deviations to M. In the rest of this note the property (1) will be demonstrated in a specific model. The investigation of the property (2) will be differed to the future publication [12].

Let us make the following identifications

$$G = SU(6)\ ,\ \ H = Sp(6)\ ,\ \text{and}\ H_w = SU_w(2) \tag{5}$$

Furthermore we specify the $SU_w(2)$ content of the fundamental representation of $SU(6) : \underline{6} = \underline{3} + 3 \times \underline{1}$ which excludes the possibility of embedding of $SU_w(2)$ into $Sp(6)$. It is convenient to choose $SU(6)$ generators (X_A) in the form of a direct product of Pauli $(\sigma_\alpha, \alpha = 0, ..., 3)$ and Gell-Mann $(\lambda_a, a = 0, ..., 8)$ matrices $X_A \sim \sigma_\alpha \otimes \lambda_a$ which will be assumed to be normalized as $Tr(X_A X_B) = \delta_{AB}$; The unbroken (H_i) and broken generators (K_m) are identified by the conditions $\sigma_2 H_i \sigma_2 = -\tilde{H}_i$ and $\sigma_2 K_m \sigma_2 = \tilde{K}_m$ where the tilda indicates the transposed matrices. The $SU_w(2)$ generators $(X_\alpha, \alpha = 1,2,3)$ are $2\sqrt{2}X_\alpha = \sigma_\alpha \otimes h + 1 \otimes \lambda_\alpha$ with $h = \frac{2}{3} + \lambda_8/\sqrt{3}$.

Note that there is a $H_c = U_c(1) \otimes SU_c(2)$ "custodial" symmetry generated by $X^c = \sigma_2 \otimes h + 1 \otimes \lambda_2$ and $X^c_\alpha = \sigma_\alpha \otimes (1-h), \alpha = 1,2,3$, which will play an important role in the following discussions. The $U_c(1)$ component represents a would be massless gauge boson and its breaking may give rise to a dynamical hierarchy as will be discussed later.

Now the calculation of the leading order effective potential $V^{(1)}(x)$ will be briefly described. The $V^{(1)}(x)$ is given in terms of the gauge boson mass square matrix $\mu^2_{\alpha\beta}(x)$ as follows

$$V^{(1)} = CTr\{\mu(x)\} \tag{6}$$

$$\mu^2_{\alpha\beta}(x) = Tr\{X_\alpha(x)K_m\}Tr\{X_\beta(x)K_m\} \tag{7}$$

$$X_\alpha(x) = l^\dagger(x)X_\alpha l(x),\ \ l(x) \in\ SU(6)/Sp(6) \tag{8}$$

where overall scales have been absorbed into the constant C. There is an equivalent form of Eq.(6) which provides a more economic framework for calculations

$$2\mu^2_{\alpha\beta}(x) = \delta_{\alpha\beta} + Tr\left\{\sigma_2\tilde{X}_\alpha(x)\sigma_2 X_\beta(x)\right\} \tag{9}$$

The task is further facilitated by a canonical parameterization of the coset element

$\ell(x)$:

$$\begin{aligned} \ell(x) &= \ell_1(\delta)\ell_2(\beta,\gamma) && (10)\\ \ell_1(\delta) &= exp(i\delta_i\sigma_i\lambda_2) \in SU(4)/Sp(4) && (11)\\ \ell_2(\beta,\gamma) &= exp\left[\frac{i}{2}(\beta_0\lambda_4+\beta_i\sigma_i\lambda_5)+\frac{i}{2}(\gamma_0\lambda_\sigma+\gamma_i\sigma_i\lambda_7)\right] && (12)\end{aligned}$$

Here the broken generators $\lambda_a, a = 1,3$ and λ_8 have been dropped since the former can be gauged away whereas the latter commutes with $SU_w(2)$. Note that the coset elements ℓ_1 and ℓ_2 are functions respectively of one (σ_δ) and two $(\sigma_\beta, \sigma_\gamma)$ quaternions and their hermitian conjugates

$$\begin{aligned} \sigma_\delta &= \vec{\delta}\vec{\sigma} && (13)\\ \sigma_\beta &= i\beta_0+\vec{\beta}\vec{\sigma} && (14)\\ \sigma_\gamma &= i\gamma_0+\vec{\gamma}\vec{\sigma} && (15)\end{aligned}$$

It is important to identify their $H_c = U_c(1)\otimes SU_c(2)$ content. One easily determines that they represent a singlet (σ_δ) and two doublets $(\sigma_\beta, \sigma_\gamma)$ of $SU_c(2)$. The $U_c(1)$ content of σ_δ is also obvious *i.e.*the $\eta \equiv \delta_2$ is neutral whereas the combination $\psi^* \equiv \delta_1 - i\delta_3$ has two unites of $U_c(1)$ charge. The $U_c(1)$ content of σ_β and σ_γ may be unraveled by considering the decomposition

$$\Phi \equiv \sigma_\beta + i\sigma_\gamma = \sum_{1=1}^{4}\Phi_i\rho_i \tag{16}$$

given in terms of the projection operators

$$\begin{aligned} &\rho_i = (1+\epsilon_i\sigma_2)/\sqrt{2}, \quad i=1,4, \quad \epsilon_1=-\epsilon_4=1\\ &\rho_i = (\sigma_1-i\epsilon_i\sigma_3)/\sqrt{2}, \quad i=2,3, \quad \epsilon_2=-\epsilon_3=1\end{aligned} \tag{17}$$

Hence one readily identifies the $SU_c(2)$ doublets $\tilde{\chi} = (\phi_1\ \phi_2)$ and $\tilde{\varphi} = (\phi_3\ \phi_4)$ as $U_c(1)$ singlets with zero and two unites of $U_c(1)$ charges respectively.

The advantage of the parametrization $(10-12)$ becomes obvious from the reduced form of the mass matrix (9):

$$2\mu^2_{\alpha\beta} = \delta_{\alpha\beta} + Tr\left(\sigma_2\tilde{X}_\alpha\sigma_2 U^\dagger X_\beta U\right) \tag{18}$$

with

$$U = \ell_1 \ell_2^2 \ell_1 \tag{19}$$

The evaluation of (18) requires only the knowledge of the 4×4 submatrix of U which may be represented as

$$U_0 \equiv (U_{ik})_{i,k=1,\ldots,4} = \sum_{\alpha,a=0}^{3} (A_{\alpha a} + iB_{\alpha a})(\sigma_\alpha \otimes \tau_a) \tag{20}$$

The calculation of the coefficients $A_{\alpha a}$ and $B_{\alpha a}$ proceeds from the formula

$$l_2^2 \equiv exp \begin{pmatrix} 0 & B \\ -B^\dagger & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{1 - zz^\dagger} & z \\ -z^\dagger & \sqrt{1 - zz^\dagger} \end{pmatrix} \tag{21}$$

where

$$B^\dagger = (\sigma_\beta^\dagger \; \sigma_\gamma^\dagger) \tag{22}$$

$$z = B(\sin\sqrt{B^\dagger B})/\sqrt{B^\dagger B} \tag{23}$$

Note that the matrix element with a square root in Eq.(21) may be easily realized due to the simple properties of the quaternions $\sigma_\beta^\dagger \sigma_\beta = \beta_0^2 + \vec{\beta}^2 \equiv \beta^2$ etc:

$$B^\dagger B = \beta^2 + \gamma^2 \equiv w^2 \tag{24}$$

$$\sqrt{1 - zz^\dagger} = cos\sqrt{BB^\dagger} = 1 + BB^\dagger(cosw - 1)/w^2 \tag{25}$$

Now the coefficients $A_{\alpha a}$ and $B_{\alpha a}$ can be calculated by a direct multiplication of the matrices in Eq.(19). The nonvanishing coefficients are found to be

$$4(A_{00} + iB_{00}) = 2c_2 w^2 - is_2 \hat{\delta}_i Tr(\Phi^\dagger \sigma_i \Phi) \tag{26}$$

$$4(A_{03} - iA_{01}) = -Tr(\Phi^\dagger \bar{\Phi}) \tag{27}$$

$$4(A_{i2} + iB_{i2}) = \left[-\delta_{ik} + (1 - c_2)\hat{\delta}_i \hat{\delta}_k\right] Tr(\Phi^\dagger \sigma_k \Phi) + 2is^2 w^2 \hat{\delta}_i \tag{28}$$

with the abbreviations $c_2 = cos2\delta, s_2 = sin2\delta, \delta^2 = \vec{\delta}^2, \hat{\delta} = \vec{\delta}/\delta$ and $\bar{\Phi} = \sigma_2 \Phi^* \sigma_2$.

We return to the mass matrix (18) with the results (20,26-28) and after some algebra arrive at the simple formula

$$2\mu_{\alpha\beta}^2 = \delta_{\alpha\beta} - Z_\alpha Z_\beta - Y_\alpha Y_\beta \tag{29}$$

Here the three-vectors $\vec{Y}$ and $\vec{Z}$ are defined in terms of the above coefficients as follows

$$w^2 Z_\alpha = -(1+cosw)A_{00}\delta_{\alpha 2} + (1-cosw)A_{\alpha 2} \tag{30}$$

$$w^2 Y_\alpha = (1+cosw)B_{\alpha 2} + (1-cosw)(A_{03}\delta_{\alpha 1} - B_{00}\delta_{\alpha 2} - A_{01}\delta_{\alpha 3}) \tag{31}$$

The eigenvalues of the matrix (29) are determined without difficulty

$$2\mu_0^2 = 1 \tag{32}$$

$$2\mu_\pm^2 = 1 - \frac{1}{2}\left\{Z^2 + Y^2) \pm \sqrt{(Z^2-Y^2)^2 + 4(ZY)^2}\right\} \tag{33}$$

This in particular implies

$$2Tr(\mu^2) = 3 - (Z^2 + Y^2) \tag{34}$$

$$4Tr(\mu^4) = 3 - 2(Z^2+Y^2) + (Z^2+Y^2)^2 - 2\left[Z^2Y^2 - (ZY)^2\right] \tag{35}$$

Hence the effective potential (6) can be deduced by a straightforward evaluation of X^2 and Y^2 ,

$$V^{(1)} \sim 1 + \frac{1}{8}\left(\frac{\sin w}{w}\right)^2 \cdot Tr\left\{\Phi^\dagger\left[(1 - c_2\sigma_2 - (1-c_2)\sigma_i\hat{\delta}_i\hat{\delta}_2\right]\Phi - s_2 \mathrm{Re}\left[\Phi^\dagger(\hat{\delta}_1 - i\hat{\delta}_3)\bar{\Phi}\right]\right\} \tag{36}$$

It is easy to verify that the result is invariant under the custodial symmetry $H_c = U_c(1) \otimes SU_c(2)$. However the invariance may be also rendered manifest by passing to the H_c multiplets $\{\eta, \psi, \chi, \varphi\}$,

$$V^{(1)} \sim 1 + \frac{1}{2}\frac{\sin^2\sqrt{|\varphi|^2 + |\chi|^2}}{(|\varphi|^2 + |\chi|^2)} \cdot \left\{s^2|\psi|^2|\chi|^2 + (1 - s^2|\psi|^2)|\varphi|^2 - 2s\left[c(\chi\epsilon\varphi)\psi + s(\chi^\dagger\varphi)\eta\psi + c.c.\right]\right\} \tag{37}$$

where

$$s = \sin\delta/\delta, \quad c = \cos\delta, \quad \delta^2 = |\psi|^2 + \eta^2 \tag{38}$$

The final step of the derivation is to verify that the expression in the curly brackets is positive. This is achieved by recasting it in a quadratic form

$$V^{(1)} \sim 1 + \frac{1}{2} \frac{\sin^2 \sqrt{|\varphi|^2 + |\chi|^2}}{(|\varphi|^2 + |\chi|^2)} \mid x_\varphi (\varphi\psi) + y_\varphi \epsilon (\varphi\psi)^* + x_\chi \chi + y_\chi \epsilon \chi^* \mid^2 \quad (39)$$

where the two dimensional vectors $\vec{r}_w = (x_w, y_w)$, $w = \varphi, \chi$ are fixed by the conditions

$$r_\varphi^2 = (s\delta)^2, \quad r_\chi^2 = 1 - r_\varphi^2 |\psi|^2 \quad (40)$$

$$\vec{r}_\varphi \cdot \vec{r}_\chi = - sc\delta, \quad \vec{r}_\varphi \times \vec{r}_\chi = - (s\delta)^2 \eta \quad (41)$$

It can be also shown that the derivation of Eq. (39) may be effected by a unitary transformation on the quaternions $(\sigma_\beta, \sigma_\gamma) \rightarrow \left(\sigma_{\beta'}, \sigma_{\gamma'}\right)$ which reduces the expression in the curly brackets to γ'^2.

Now we are in a position to demonstrate how the first necessary condition for dynamical hierarchies may be realized in our model (see Eq(1)).The effective potential (37) attains its minimum on two submanifolds $M_i \subset SU(6)/Sp(6), i = 1, 2$,

$$M_1 : |s\psi\chi|^2 + \left(1 - |s\psi|^2\right) |\varphi|^2 - 2s \left[c\psi (\chi\epsilon\varphi) + s\eta\psi \left(\chi^\dagger \varphi\right) + \text{c.c.}\right] = 0 \quad (42)$$

$$M_2 : |\varphi|^2 + |\chi|^2 = [(2n + 1)\pi]^2 \quad (43)$$

The degeneracy of the minima will be lifted by the higher order contribution $V^{(2)}(x)$ to the effective potential $V(x)$. (c.f. Eq. (2)). It will be assumed that at the minimum of $V^{(2)}(x \in M_i)$ the parameter ψ vanishes $\psi = \psi_m = 0$; A nonvanishing $\psi_m \neq 0$ would render the $U_c(1)$ gauge boson massive without leading to a dynamical hierarchy. Then the condition (41) implies $\varphi = \varphi_m = 0$ and the gauge boson spectrum follows from Eqs. (30-33).

$$- Z_\alpha / \cos 2\delta = Y_\alpha \delta / \eta \sin 2\delta = \delta_{\alpha 2}, \quad \psi = \varphi = 0 \quad (44)$$

$$\mu_0 = \mu_- = \frac{1}{2}, \quad \mu_+ = 0. \quad (45)$$

Observe that one of the gauge bosons remains massless for an arbitrary value of $\chi = \chi_m$. This is because the $\{\chi_m \neq 0 | \psi_m = \varphi_m = 0\}$ triggers a breakdown $SU_c(2) \otimes U_c(1) \rightarrow U_g(1)$ and surviving $U_g(1)$ generated by $(\sigma_2 h + \lambda_2) + \sigma_2 (1 - h)$ represents the massless mode in the spectrum (45).

Returning to Eq.(37), one finds that the expansion about $\psi_m = \varphi_m = 0, \ \ \chi_m \neq 0$ does not lead to the condition (1) unless the minimum is on the intersection of the manifolds M_1 and M_2, i.e., $|\chi_m|^2 = |(2n+1)\pi|^2$. Therefore we turn to the second possibility (42). The "critical" points of interest are $\psi_m = \chi_m(\varphi_m) = 0$ which imply the same mass spectrum as (45) and moreover lead to the required behavior of the effective potential (c.f. Eq. (1)).

$$V^{(1)}(\psi_m = \chi_m = 0) \ \sim \ |\chi|^4 \tag{46}$$

$$V^{(1)}(\psi_m = \varphi_m = 0) \ \sim \ |\varphi|^6 \tag{47}$$

The distinct behaviors (46) and (47) reflect different breaking patterns of the custodial symmetry $H_c \to U_c(1)$ or $H_c \to U_g(1)$. Evidently, the $U_c(1)$ or $U_g(1)$ symmetry will break down and the dynamical hierarchy will result if the condition (2) holds.

It should be re-emphasized that the assumption $\psi_m = \chi_m(\varphi_m) = 0$ underlying the above results depends on the higher order effective potential $V^{(2)}$. It is encouraging to note that the component $V_I^{(2)}$ of $V^{(2)}$ arising from the vector boson mass matrix supports the relation (44). Indeed the $V_I^{(2)} \sim -Tr(\mu^4)$[11] given by Eq. (35) and constrained by the condition $(Z^2+Y^2)_{M_2} = 1$ attains its minimum at $\vec{Z}||\vec{Y}$ which is consistent with the relation (44).

In summary, we have demonstrated in a specific model, that a leading order superweak gauge interactions can generate a pure (without mass term) self interaction of the composite Higgs boson with quantum numbers of the massless gauge boson. Furthermore it has been argued that under certain conditions depending on higher order effects the Higgs can condense rendering the gauge boson massive and leading to a dynamical gauge hierarchy. Obviously it is imperative to identify a class of models which possess the properties (c.f.Eq.(1,2)) necessary for a general realization of the program proposed in this note.

Acknowledgements

This work was begun at CERN and completed at Fermilab. I would like to acknowledge warm and generous hospitality of these institutions. In particular, I am deeply grateful to my hosts Daniele Amati, John Ellis, Estia Eichten and Chris Quigg.Also it is a pleasure to thank Estia Eichten and Shimon Yankielowitcz for stimulating conversations.

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